On Almost Soft$\beta$-Continuous Functions

Saif Z. Hameed, Ahmad Al-Omari

Al Mustansiriya University, college of basic education, Department of Mathematics

saif.zuhar.edbs@uomustansiriya.edu.iq

Al al-Bayt University, Faculty of Sciences, Department of Mathematics P.O. Box 130095, Mafraq 25113, Jordan

omarimutah1@yahoo.com

Abstract

Let $X$ be an initial universal set and $P(X)$ the power set of $X$, $A$ a set of parameters. A pair $(F, A)$, where $F$ is a map from $A$ to $P(X)$, is called a soft set over $X$. A soft set $(F, A)$ of a soft topological space $(X, \tau, A)$ is said to be soft $\beta$-open if $(F, A) \subseteq \text{scl}(\text{sint}(\text{scl}(F, A)))$. Then some related properties and characterizations of soft $\beta$-open sets have been discussed. By using the notion of soft $\beta$-open sets several classes of functions called soft $\beta$-continuous, almost soft $\beta$-continuous, weakly soft $\beta$-continuous. Also, several properties of these functions were studied and investigated.

Keyword: Soft $\beta$-open, Soft $\beta$-continuous, weakly Soft $\beta$-continuous, almost Soft $\beta$-continuous.

1. Introduction

The concept of soft sets was first introduced by Molodtsov [8] in 1999 who began to develop the basics of corresponding theory as a new approach to modeling uncertainties. In [18] and [9], Molodtsov successfully applied the soft theory in several directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and measure theory.

In recent years, an increasing number of papers have been written about soft sets theory and its applications in various fields by P. K. Maji et al [22] and I. Zorlutuna et al [13]. Shabir and Naz [19] introduced the notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameters. In addition, Maji et al. [22] proposed several operations on soft sets, and some basic properties of these operations have been revealed so far.

In general topology, the concept of $\beta$-open sets was introduced by Abd El-Monsef et al [1] and $\beta$-open sets have been referred to as semipreopen by Andrijevic [7]. In [12] I. Arockiarani and A. Arokialancy this concept has been generalized to soft setting.

Recently, Borsike and Dob o s [14] have introduced the notion of almost quasi-continuity. Also, T. Noiri and V. Popa [26] showed that almost quasi-continuity is equivalent to $\beta$-continuity. Quite recently, T. Noiri [24] have introduced the notions of weak $\beta$-continuity and almost $\beta$-continuity in topological spaces. Weak $\beta$-continuity has been investigated by T. Noiri and V. Popa [27]. A. A. Nasef and T. Noiri [1] have studied fundamental properties of almost $\beta$-continuous functions.

2.Preliminaries

Let $(X, \tau, A)$ be a soft topological spaces, where $A$ be any set of parameters. The soft closure (resp. soft interior) sets are denoted by $\text{scl}(F, A)$ (resp. $\text{sint}(F, A)$). A subset $(F, A)$ is said to be soft $\beta$-open [12](resp. Soft $\alpha$-open [17]. Soft preopen [12], Soft semi-open [6]) if $(F, A) \subseteq \text{scl}(\text{sint}(\text{scl}(F, A)))$ (resp. $(F, A) \subseteq \text{sint}(\text{scl}(\text{sint}(F, A)))$ , $(F, A) \subseteq \text{sint}(\text{scl}(F, A))$ , $(F, A) \subseteq \text{scl}(\text{sint}(F, A))$ ). The complement of a soft $\beta$-open set is called soft $\beta$-closed. Soft $\beta$-closure and soft $\beta$-interior of a soft set are defined as follows.

**Definition 2.1**[2, 15, 5]. Let $(X, \tau, A)$ be a soft topological space and let $(F, A)$ be a soft set over $X$.

1. Soft $\alpha$-interior of a soft $(F, A)$ in $X$ is denoted by $\text{sint}(F, A) = \mathcal{O}(O, A) := (O, A)$ is a soft $\alpha$-open set and $(O, A) \subseteq (F, A)$.
2. Soft $\alpha$-closure of a soft $(F, A)$ in $X$ is denoted by $\text{scl}(F, A) = \mathcal{O}(O, A) := (O, A)$ is a soft $\alpha$-open set and $(O, A) \subseteq (F, A)$.


\( s\text{accl}((F, A)) = \tilde{\cap}\{ (F, E) : (F, E) \text{ is a soft } \alpha \text{–closed set and } (F, A) \subset (F, E) \}\).

3. Soft pre-interior of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{Pint}((F, A)) = \tilde{\cap}\{ (O, A) : (O, A) \text{ is a soft preopen set and } (O, A) \subset (F, A) \}\].

4. Soft pre-closure of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{Pcl}((F, A)) = \tilde{\cap}\{ (F, E) : (F, E) \text{ is a soft preclosed set and } (F, A) \subset (F, E) \}\].

5. Soft semi-interior of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{Sint}((F, A)) = \tilde{\cap}\{ (O, A) : (O, A) \text{ is a soft semi-open set and } (O, A) \subset (F, A) \}\].

6. Soft semi-closure of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{Scl}((F, A)) = \tilde{\cap}\{ (F, E) : (F, E) \text{ is a soft semi-closed set and } (F, A) \subset (F, E) \}\].

7. Soft \(\beta\)-interior of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{bint}((F, A)) = \tilde{\cap}\{ (O, A) : (O, A) \text{ is a soft } \beta \text{–open set and } (O, A) \subset (F, A) \}\].

8. Soft \(\beta\)-closure of a soft set \((F, A)\) in \(X\) is denoted by
\[ s\text{bcl}((F, A)) = \tilde{\cap}\{ (F, E) : (F, E) \text{ is a soft } \beta \text{–closed set and } (F, A) \subset (F, E) \}\].

Clearly \(s\text{accl}((F, A))\) (resp., \(s\text{Pcl}((F, A))\), \(s\text{Scl}((F, A))\), \(s\beta\text{cl}((F, A))\)) is the smallest soft \(\alpha\)-closed (resp. preclosed, semi-closed, \(\beta\)-closed) set over \(X\) which contains \((F, A)\) and \(s\text{aint}((F, A))\) (resp. \(s\text{Pint}((F, A))\), \(s\text{Sint}((F, A))\), \(s\beta\text{int}((F, A))\)) is the largest soft \(\alpha\)-open (resp. preopen, semi-open, \(\beta\)-open) set over \(X\) which is contained in \((F, A)\).

We will denote the family of all soft \(\alpha\)-open (resp., preopen, semi-open, \(\beta\)-open) sets (resp., soft \(\alpha\)-closed, preclosed, semi-closed, \(\beta\)-closed) sets of a soft topological space by \(S\alpha\text{OS}(X)\) (resp., \(S\text{POS}(X)\), \(S\text{SOS}(X)\), \(S\beta\text{OS}(X)\)) (resp., \(S\alpha\text{CS}(X)\), \(S\text{PCS}(X)\), \(S\text{SCS}(X)\), \(S\beta\text{CS}(X)\)). The family \(\{(U, A) \in S\beta\text{OS}(X) : x \in (U, A)\}\) is denoted by \(S\beta\text{OS}(X, x)\) where \(x\) is a point of \(X\). \(S\text{SOS}(X, x)\), \(S\text{POS}(X, x)\) and \(S\alpha\text{OS}(X, x)\) are defined similarly.

**Definition 2.2.**[10] Let \((X, \tau, A)\) be a soft topological space, and \((F, A)\) is soft open set in \(X\). A point \(x \in X\) is called a soft \(\delta\)-cluster (resp., \(\theta\)-cluster) point of \((F, A)\) if \((F, A) \cap \text{int}(\text{scl}(U, A)) \neq \emptyset\) (resp., \((F, A) \cap \text{scl}(U, A)) \neq \emptyset\) for each soft open set \((U, A)\) containing \(x\).

The set of all soft \(\delta\)-cluster (resp., \(\theta\)-cluster) point of \((F, A)\) is called the soft \(\delta\)-closure (resp., soft \(\theta\)-closure) of \((F, A)\) and is denoted by \(s\delta\text{cl}((F, A))\) (resp., \(s\theta\text{cl}((F, A))\)).

If \(s\delta\text{cl}((F, A)) = (F, A)\) (resp., \(s\theta\text{cl}((F, A)) = (F, A)\)), then \((F, A)\) is said to be soft \(\delta\)-closed (resp., soft \(\theta\)-closed). The complement of a soft \(\delta\)-closed (resp., soft \(\theta\)-closed) set is said to be soft \(\delta\)-open (resp., soft \(\theta\)-open).

**Theorem 2.3.**[16] Let \((F, A)\) be soft set of \(X\). Then
\[ s\beta\text{int}((F, A)) = (F, A) \cap \text{scl}(\text{int}(\text{scl}((F, A)))). \]

**Proof.**

We observe that \((F, A) \cap \text{scl}(\text{int}(\text{scl}((F, A)))) \subset \text{scl}(\text{int}(\text{scl}((F, A))))\)

Hence \((F, A) \cap \text{scl}(\text{int}(\text{scl}((F, A)))) \in S\beta\text{OS}(X)\) and thus
(\(F, A\)) \(\rightsquigarrow\) \(scl((\text{scl}(scl((F, A)))))) \subseteq \text{sflint}((F, A))\). On the other hand, since \(\text{sflint}((F, A))\) is \(\beta -\text{open}\), we have \(\text{sflint}((F, A)) \subseteq \text{scl}((\text{scl}(\text{sflint}((F, A)))))) \subseteq \text{scl}((\text{scl}(scl((F, A))))))\) and hence \(\text{sflint}((F, A)) \subseteq (F, A) \rightsquigarrow \text{scl}((\text{scl}(scl((F, A))))))\).

**Theorem 2.4.** For any soft set \((F, A)\) of a soft topological space \((X, \tau, A)\) the following condition are equivalent:

1. \((F, A) \in \text{SfSO}(X)\),
2. \((F, A) \subseteq \text{scl}((\text{scl}(scl((F, A))))))\),
3. \(\text{scl}((F, A)) \in \text{SRCS}(X)\).

**Corollary 2.5.**

1. If \((H, A)\) is a soft \(\alpha\)-open set and \((F, A)\) is a soft \(\beta\)-open set then \((H, A) \rightsquigarrow (F, A)\) is a soft \(\beta\)-open set.
2. If \((K, A)\) is a soft \(\alpha\)-closed set and \((F, A)\) is a soft \(\beta\)-closed set then \((K, A) \rightsquigarrow (F, A)\) is a soft \(\beta\)-closed set.

### 3. On Almost Soft \(\beta\)-Continuous Functions

We will introduce some definitions and properties of almost \(\text{soft } \beta -\text{continuous}\) functions and we obtain many characterizations of almost \(\text{soft } \beta -\text{continuous}\) functions and deal with several properties concerning soft \(\beta\)-continuous, almost soft \(\beta\)-continuous and weakly soft \(\beta\)-continuous. sectionSome definitions and properties.

**Lemma 3.1.** Let \((H, A)\) be a soft subset in a soft topological space over \(X\). Then

1. \(\text{sflcl}((X \setminus (H, A))) = X \setminus \text{sflint}((H, A))\).
2. If \(x \in \text{sflcl}((H, A)) \Leftrightarrow (H, A) \rightsquigarrow (U, A) \neq \phi\) for each \((U, A) \in \text{SfSO}(X, x)\).

**Lemma 3.2.** \(\text{sScl}(H, A) = \text{sint}((\text{scl}(scl((H, A))))))\) for every soft preopen set \((H, A)\) of soft topological space \((X, \tau, A)\).

**Definition 3.3.** Let \(f : X \rightarrow Y\) be a function then \(f\) is said to be:

1. **Weakly soft \(\beta\)-continuous** (denoted by \(w.s.\beta.c\)) at \(x \in X\) if for each softopen set \((G, A)\) of \(Y\) containing \(f(x)\), there exists \((F, A) \in \text{SfSO}(X, x)\) such that \(f((F, A)) \subseteq \text{cl}((G, A))\).
2. **Almost soft \(\beta\)-continuous** (denoted by \(a.s.\beta.c\)) at \(x \in X\) if for each soft open set \((G, A)\) of \(Y\) containing \(f(x)\), \(\exists (F, A) \in \text{SfSO}(X, x)\) such that \(f((F, A)) \subseteq \text{sint}((\text{scl}(scl((G, A))))))\).

**Remark 3.4.** Know that soft \(\beta\)-continuity implies almost soft \(\beta\)-continuity, a almost soft \(\beta\)-continuity implies weakly soft \(\beta\)-continuity. However, in general the converses are not true.

**Example 3.5.** Let \(X = \{h_1, h_2\}\), \(A = \{e_1, e_2\}\) define

\[
(F_1, A) = \{(e_1, \phi), (e_2, \phi)\} = \tilde{\phi},
\]

\[
(F_2, A) = \{(e_1, \phi), (e_2, \{h_1\})\}, \quad (F_3, A) = \{(e_1, \phi), (e_2, \{h_2\})\},
\]

\[
(F_4, A) = \{(e_1, \phi), (e_2, X)\}, \quad (F_5, A) = \{(e_1, \{h_1\}), (e_2, \phi)\},
\]

\[
(F_6, A) = \{(e_1, \{h_1\}), (e_2, h_1)\}, \quad (F_7, A) = \{(e_1, \{h_1\}), (e_2, h_2)\},
\]

\[
(F_8, A) = \{(e_1, \{h_1\}), (e_2, X)\}, \quad (F_9, A) = \{(e_1, \{h_2\}), (e_2, \phi)\},
\]

\[
(F_{10}, A) = \{(e_1, \{h_2\}), (e_2, \{h_1\})\}, \quad (F_{11}, A) = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}.
\]
\[ (F_{12}, A) = \{(e_1, \{h_2\}), (e_2, X)\}, \quad (F_{13}, A) = \{(e_1, X), (e_2, \phi)\}, \]
\[ (F_{14}, A) = \{(e_1, X), (e_2, \{h_1\})\}, \quad (F_{15}, A) = \{(e_1, X), (e_2, \{h_2\})\}, \]
\[ (F_{16}, A) = \{(e_1, X), (e_2, X)\} = \tilde{X}. \]

are all soft sets on universal set \( X \) under the parameter \( A \).
\[ \tau = \{(F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A)\} \]is a soft topology over \( X \).
Soft open sets are: \( (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A) \).
Soft closed set are: \( (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A) \).
Soft \( \beta \)-open sets are: \( (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A) \).
Soft \( \beta \)-closed sets are: \( (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A) \).

Now, let us take \( Y = \{y_1, y_2\} \), \( K = \{k_1, k_2\} \) define \( \nu = \{\tilde{Y}, \tilde{\phi}, (F, K)\} \) where \( (F, K) = \{(k_1, \phi), (k_2, \{y_2\})\} \).

Define \( f : X \rightarrow Y \) and \( p : A \rightarrow K \) as \( f(h_1) = y_2, f(h_2) = y_1 \), \( p(e_1) = k_1, p(e_2) = k_2 \). Then \( \text{sint}((F, K)) = \tilde{Y} \) and we can find soft \( \beta \)-open set \((G, A)\) such that \( f((G, A)) \supseteq \text{sint}((F, K)) \). Then \( f \) is almost soft \( \beta \)-continuous functions, but not soft \( \beta \)-continuous functions.

**Remark 3.6.** The Cardinality of \( SS(X, A) \) is given by \( n(SS(X, A)) = 2^{n(X) n(A)} \).

**Example 3.7.** In Example 3.5, \( n(SS(X, A)) = 2^{2 \times 2} = 16 \).
And, if \( X = \{a, b, c\}, A = \{e_1, e_2\} \) then \( n(SS(X, A)) = 2^{3 \times 2} = 2^6 = 64 \).

**Theorem 3.8.** Let \( f : (X, \tau, A) \rightarrow (Y, \nu, A) \) be a function, then the following are equivalent:
1. \( f \) is a.s.\( \beta \)-c. ;
2. \( \forall x \in X \) and each \( (F, A) \in \nu \) containing \( f(x) \), \( \exists (G, A) \in \text{S\beta OS}(X) \) containing \( x \) such that \( f(G, A) \supseteq \text{sint}((\text{scl}(F, A))) ; \)
3. \( f^{-1}((K, A)) \in \text{S\beta CS}(X) \) \( \forall (K, A) \in \text{SRC}(Y, A) ; \)
4. \( f^{-1}((F, A)) \in \text{S\beta OS}(X) \) \( \forall (F, A) \in \text{SRO}(Y, A) ; \)

**Theorem 3.9.** Let \( f : X \rightarrow Y \) be a function, then the following are equivalent:
1. \( f \) is a.s.\( \beta \)-c. at \( x \in X ; \)
2. For all soft neighbourhood \((G, A)\) of \( f(x) \), \( x \in \text{scl}(\text{sint}(\text{scl}(f^{-1}(\text{scl}(G, A)))))) ; \)
3. For all soft neighbourhood \((G, A)\) of \( f(x) \) and each soft neighbourhood \((U, A)\) of \( x \), there exists a nonempty soft open set \((F, A) \supseteq (U, A) \) such that \((F, A) \supseteq \text{cl}(f^{-1}(\text{scl}(G, A))) ; \)
4. For all soft neighbourhood \((G, A)\) of \( f(x) \), there exists \((U, A) \in \text{SSOS}(X, x) \) such that \((U, A) \supseteq \text{scl}(f^{-1}(\text{scl}(G, A))) \).

**Proof.**

(1) \( \Rightarrow \) (2). Let \((G, A)\) be a soft neighbourhood of \( f(x) \). Then there exists \((U, A) \in \text{S\beta OS}(X, x) \) such that \( f((U, A)) \supseteq \text{scl}(G, A) \). Then, \((U, A) \supseteq f^{-1}(\text{scl}(G, A)) \). Since \((U, A)\) is soft \( \beta \)-open.

\( x \in (U, A) \supseteq \text{scl}(\text{sint}(\text{scl}(U, A))) \subseteq \text{scl}(\text{sint}(f^{-1}(\text{scl}(G, A)))) \).
(2) $\Rightarrow$ (3). Let $(G, A)$ be a soft neighbourhood of $f(x)$, $(U, A)$ an open soft set of $X$ containing $x$. Since $x \in scl(sint(scl(f^{-1}(sScl((G, A))))))$, we have

$$(U, A) \cap sint(scl(f^{-1}(sScl((G, A)))))) \neq \emptyset.$$ Put $(F, A) = (U, A) \cap sint(scl(f^{-1}(sScl((G, A))))))$, then $(F, A)$ is a nonempty soft open set, $(F, A) \subsetneq (U, A)$ and

$$(F, A) \subsetneq sint(scl(f^{-1}(sScl((G, A)))))) \subsetneq scl(f^{-1}(sScl((G, A))))).$$

(3) $\Rightarrow$ (4). Let $(U_x, A)$ set of all soft open sets containing $x$. For any soft open set $(U, A) \in (U_x, A)$ and any soft neighbourhood $(G, A)$ of $f(x)$, there exists a soft open set $(F, A)_{(U, A)} \subsetneq (U, A)$, such that

$$(F, A)_{(U, A)} \subsetneq scl(f^{-1}(sScl((G, A))))).$$

Then by Theorem 2.3 we have

$$(U, A) \cap sint(scl(f^{-1}(sScl((G, A)))))) \subsetneq f^{-1}(sScl((G, A)))).$$ $

\text{Therefore, put}

\medskip

$$f((U, A)) \subsetneq scl(scl(f^{-1}(sScl((G, A))))).$$

Theorem 3.10. Let $f : X \rightarrow Y$ be a function, then the following are equivalent:

1. $f$ is a.s. $\beta c.$;
2. $s\bar{f}k\bar{c}(f^{-1}(scl(sint(scl(K, A))))) \subsetneq f^{-1}(scl((K, A))))$ for every soft subset $(K, A)$ of $Y$;
3. $s\bar{f}k\bar{c}(f^{-1}(scl(sint((H, A))))) \subsetneq f^{-1}((H, A))$ for every soft closed set $(H, A)$ of $Y$;
4. $s\bar{f}k\bar{c}(f^{-1}(scl((G, A)))) \subsetneq f^{-1}((scl((G, A))))$ for every soft open set $(G, A)$ of $Y$;
5. $f^{-1}((G, A)) \subsetneq s\bar{f}k\bar{c}(f^{-1}(sScl((G, A))))$ for every soft open set $(G, A)$ of $Y$;
6. $f^{-1}((G, A)) \subsetneq scl(scl(f^{-1}(sScl((G, A))))))$ for every soft open set $(G, A)$ of $Y$.

Proof.

(1) $\Rightarrow$ (2). Let $(K, A)$ be a soft subset of $Y$ and $x \in X \setminus f^{-1}(cl((K, A))))$. Then $f(x) \in Y \setminus scl((K, A))$ and there exists a soft open set $(G, A)$ containing $f(x)$ such that $(G, A) \not\supseteq (K, A)$, hence

$$scl(scl(f^{-1}(sScl((K, A)))))) \subsetneq f^{-1}(cl((K, A))))$$

(2) $\Rightarrow$ (3). Let $(H, A)$ be any soft closed set in $Y$. Then

$$s\bar{f}k\bar{c}(f^{-1}(scl(sint((H, A))))) = s\bar{f}k\bar{c}(f^{-1}(scl(sint(int((H, A)))))))$$

$$\subsetneq f^{-1}(cl((H, A)))) \subsetneq f^{-1}((H, A))).$$
For any soft open set \((G, A)\) in \(Y\), \(scl((G, A))\) is softregular closed set in \(Y\), we have;
\[
\text{sfkl}(f^{-1}(cl((G, A)))) = \text{sfkl}(f^{-1}(cl(int(cl((G, A)))))) \supseteq f^{-1}(cl((G, A))).
\]

Let \((G, A)\) be a soft open set in \(Y\). Then \(Y \setminus scl((G, A))\) is soft open in \(Y\) and by using (Lemma 3.1 and 3.2), we have;
\[
X - sfint(f^{-1}(sScI((G, A)))) = sfkl(f^{-1}(Y \setminus sint(scl((G, A))))). \subseteq \nabla X \setminus f^{-1}((G, A)).
\]

Therefore, we obtain
\[
f^{-1}((G, A)) \supseteq sfint(f^{-1}(sScI((G, A)))).
\]

Let \((G, A)\) be a soft open set in \(Y\). By using Theorem 2.3, we obtain;
\[
f^{-1}((G, A)) \supseteq sfint(f^{-1}(sScI((G, A)))) \supseteq scl(sint(scl(f^{-1}(sScI((G, A)))))).
\]

Let \(x\) be a soft point in \(X\) and \((G, A)\) be a soft open set in \(Y\) containing \(f(x)\). Then \(x \in f^{-1}((G, A)) \supseteq scl(sint(scl(f^{-1}(sScI((G, A)))))).\)

From Theorem 3.9 that \(f\) is as.\(\beta\)c. at any point \(x\) of \(X\). Therefore, \(f\) is as.\(\beta\)c.

**Theorem 3.11.** Let \(f : X \rightarrow Y\) be a function, then the following are equivalent:

1. \(f\) is as.\(\beta\)c.;
2. \(sfkl(f^{-1}((G, A))) \supseteq f^{-1}(scl((G, A))) \forall (G, A) \in S\ Beta\ OS(Y);\)
3. \(sfkl(f^{-1}((G, A))) \supseteq f^{-1}(scl((G, A))) \forall (G, A) \in S\ SOS(Y);\)
4. \(f^{-1}((G, A)) \supseteq sfint(f^{-1}(scl((G, A)))) \forall (G, A) \in S\ POS(Y).\)

**Proof.**

(1) \(\Rightarrow\) (2). Let \((G, A)\) be a soft \(\beta\)-open set in \(Y\). From Theorem 2.4, \(cl((G, A))\) is soft regular closed set in \(Y\). Since \(f\) is as.\(\beta\)c., \(f^{-1}(cl((G, A)))\) is soft \(\beta\)-closed in \(X\) Theorem 3.8. Therefore, we obtain
\[
sfkl(f^{-1}((G, A))) \supseteq f^{-1}(scl((G, A))).
\]

(2) \(\Rightarrow\) (3). This is obvious since \(S\ SOS(Y) \supseteq S\ Beta\ OS(Y).\)

(3) \(\Rightarrow\) (1). Let \((F, A)\) be any soft regular closed set in \(Y\). Then \((F, A)\) is soft semi-open in \(Y\) and hence
\[
sfkl(f^{-1}((F, A))) \supseteq f^{-1}(cl((F, A))) = f^{-1}((F, A)).
\]

We show that \(f^{-1}((F, A))\) is soft \(\beta\)-closed. Therefore, \(f\) is as.\(\beta\)c. Theorem 3.8.

(1) \(\Rightarrow\) (4). Let \((G, A)\) be a soft preopen set in \(Y\). Then \((G, A) \supseteq sint(scl((G, A)))\), \(sint(scl((G, A)))\) are soft regular open set in \(Y\). Since \(f\) is as.\(\beta\)c., \(f^{-1}(sint(scl((G, A))))\) is soft \(\beta\)-open in \(X\) Theorem 3.8. Hence we obtain that
\[
f^{-1}((G, A)) \supseteq f^{-1}(sint(scl((G, A)))) \supseteq sfint(f^{-1}(scl((G, A))))).
\]

Therefore, \(f^{-1}((G, A))\) is soft \(\beta\)-open in \(X\) and hence \(f\) is as.\(\beta\)c. Theorem 3.8.

**Lemma 3.12.** For soft subset in soft space \(Y\), the following hold:

1. \(scl((G, A)) = scl((G, A))\) for all \((G, A) \in S\ Beta\ OS(Y).\)
2. \(spcl((G, A)) = scl((G, A))\) for all \((G, A) \in S\ SOS(Y).\)
Corollary 3.13. Let \( f: X \to Y \) be a function, then the following are equivalent:

1. \( f \) is a.s.\( \beta \)-c.c.;
2. \( \beta \text{cl}(f^{-1}((G, A))) \supseteq f^{-1}(s\text{acl}(((G, A)))) \) for all \( (G, A) \in S\beta\text{OS}(Y) \);
3. \( \beta \text{cl}(f^{-1}((G, A))) \supseteq f^{-1}(s\text{pcl}(((G, A)))) \) for all \( (G, A) \in S\text{SOS}(Y) \);
4. \( f^{-1}((G, A)) \supseteq \beta\text{int}(f^{-1}(s\text{Sc}(((G, A)))) \) for all \( (G, A) \in S\text{POS}(Y) \).

Theorem 3.14. Let \( f: (X, \tau, A) \to (Y, \nu, A) \) be a function, then the following are equivalent:

1. \( f \) is a.s.\( \beta \)-c.c.;
2. \( f(\beta\text{cl}((F, A))) \supseteq \beta\text{cl}(f((F, A))) \) for every soft set \( (F, A) \) of \( X \);
3. \( \beta\text{cl}(f^{-1}((G, A))) \supseteq f^{-1}(s\text{cl}(((G, A)))) \) for every soft set \( (G, A) \) of \( Y \);
4. \( f^{-1}((H, A)) \in S\beta\text{CS}(X) \) for every soft \( \delta \)-closed set of \( (Y, \nu, A) \);
5. \( f^{-1}((K, A)) \in S\beta\text{OS}(X) \) for every soft \( \delta \)-open set \( (K, A) \) of \( (Y, \nu, A) \).

Proof.

(1) \( \Rightarrow \) (2). Let \( (F, A) \) be a soft set in \( X \). Since \( \beta\text{cl}(f((F, A))) \) is soft \( \delta \)-closed in \( Y \), it denoted by \( \tilde{\gamma}(\{F_{\alpha}, E\} : (F_{\alpha}, E) \in \text{SRCS}(Y), \alpha \in \nabla \}, \) where \( \nabla \) is an index soft set. By Theorem 3.8, we have \( (F, A) \supseteq f^{-1}(s\text{cl}(f((F, A)))) \) \( \Rightarrow \) \( \tilde{\gamma}(f^{-1}(F, \alpha) : \alpha \in \nabla) \in S\beta\text{CS}(X) \), hence \( \beta\text{cl}((F, A)) \supseteq f^{-1}(s\text{cl}(f((F, A)))) \). Therefore, we obtain \( f(\beta\text{cl}((F, A))) \supseteq (s\text{cl}(f((F, A)))) \).

(2) \( \Rightarrow \) (3). Let \( (G, A) \) be a soft set in \( Y \). We have

\[ f(\beta\text{cl}(f^{-1}((G, A)))) \supseteq \beta\text{cl}(f(f^{-1}((G, A)))) \supseteq \beta\text{cl}(f((G, A))) \]

and hence \( \beta\text{cl}(f^{-1}(((G, A))) \supseteq f^{-1}(\beta\text{cl}(f((G, A)))) \).

(3) \( \Rightarrow \) (4). Let \( (H, A) \) be a soft \( \delta \)-closed set of \( (Y, \nu, A) \). We have

\[ \beta\text{cl}(f^{-1}((H, A))) \supseteq f^{-1}(\beta\text{cl}(f((H, A)))) = f^{-1}((H, A)) \]

and hence \( f^{-1}((H, A)) \) is soft \( \beta \)-closed in \( (X, \tau, A) \).

(4) \( \Rightarrow \) (5). Let \( (K, A) \) be a soft \( \delta \)-open set in \( (Y, \nu, A) \). We have

\[ f^{-1}(Y - (K, A)) = X - f^{-1}((K, A)) \in S\beta\text{CS}(X) \]

and hence \( f^{-1}((K, A)) \in S\beta\text{OS}(X) \).

(5) \( \Rightarrow \) (1). Let \( (K, A) \) be a soft regular open set in \( Y \). Since \( (K, A) \) is soft \( \delta \)-open in \( Y \), \( f^{-1}((K, A)) \in S\beta\text{OS}(X) \) and hence by Theorem 3.8 \( f \) is a.s.\( \beta \)-c.c.

Theorem 3.15. The following are equivalent for a function \( f: X \to Y \):

1. \( f \) is w.s.\( \beta \)-c.c.;
2. \( f(\beta\text{cl}((F, A))) \supseteq \beta\text{cl}(f((F, A))) \) for each soft set \( (F, A) \) of \( Y \);
3. \( \beta\text{cl}(f^{-1}((G, A))) \supseteq f^{-1}(\beta\text{cl}(((G, A)))) \) for each soft set \( (G, A) \) of \( Y \);
4. \( \beta\text{cl}(f^{-1}(\text{int}(\beta\text{cl}(((G, A)))))) \supseteq f^{-1}(\beta\text{cl}(((G, A)))) \) for every soft set \( (G, A) \) of \( Y \).

4. Properties of almost soft \( \beta \)-continuous functions

Definition 4.1. A function \( f: X \to Y \) is said to be soft faintly \( \beta \)-continuous if for all \( x \in X \) and each soft \( \theta \)-open
set \((G, A)\) of \(Y\) containing \(f(x)\), there exists \((K, A)\) such that \(f((K, A)) \subset (G, A)\).

**Theorem 4.2.** The following are equivalent for a function \(f : (X, \tau, A) \to (Y, \nu, A)\):

1. \(f\) is soft faintly \(\beta\)-continuous;
2. \(f : (X, \tau, A) \to (Y, \nu, A)\) is soft \(\beta\)-continuous;
3. \(f^{-1}(F, A) \in S\beta OS(X, \nu, A)\) for every set \((F, A) \in \nu, A\);
4. \(f^{-1}(F, A)\) is soft \(\beta\)-closed in \(X\) for every \(\theta\)-closed set \((F, A)\) in \((Y, \nu, A)\).

**Theorem 4.3.** The implications \((1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5)\) hold for the following properties of a function \(f : X \to Y\):

1. \(f\) is soft \(\beta\)-continuous;
2. \(f^{-1}(s\beta cl((K, A)))\) is soft \(\beta\)-closed in \(X\) for every soft subset \((K, A)\) of \(Y\);
3. \(f\) is a.s. \(\beta\)-continuous;
4. \(f\) is w.s. \(\beta\)-continuous;
5. \(f\) is soft faintly \(\beta\)-continuous.

**Proof.**

\((1) \Rightarrow (2)\). Since \(s\beta cl((K, A))\) is soft closed in \(Y\), for every soft subset \((K, A)\) of \(Y\), by Theorem 4.2, \(f^{-1}(s\beta cl((K, A)))\) is soft \(\beta\)-closed in \(X\).

\((2) \Rightarrow (3)\). For any soft subset \((K, A)\) of \(Y\), \(f^{-1}(s\beta cl((K, A)))\) is soft \(\beta\)-closed in \(X\), hence we have \(s\beta cl(f^{-1}((K, A))) \subset s\beta cl(f^{-1}(s\beta cl((K, A)))) = f^{-1}(s\beta cl((K, A)))\).

It follows from Theorem 3.14 that \(f\) is a.s. \(\beta\)-continuous.

\((3) \Rightarrow (4)\). This is obvious.

\((4) \Rightarrow (5)\). Let \((H, A)\) be a \(\theta\)-closed set in \(Y\). From Theorem 3.15 that \(s\beta cl(f^{-1}((H, A))) \subset f^{-1}(s\beta cl((H, A))) = f^{-1}((H, A))\). Therefore, \(f^{-1}((H, A))\) is soft \(\beta\)-closed in \(X\), then \(f\) is soft faintly \(\beta\)-continuous by Theorem 4.2.

Suppose that \(Y\) is soft regular. We prove that \((5)\) implies \((1)\). Let \((G, A)\) be a soft open set of \(Y\). Since \(Y\) is soft regular, \((G, A)\) is soft \(\theta\)-open in \(Y\). By soft faint \(\beta\)-continuity of \(f\), \(f^{-1}((G, A))\) is soft \(\beta\)-open in \(X\). Theorem 4.2. Therefore, \(f\) is soft \(\beta\)-continuous.

**Definition 4.4.** A function \(f : X \to Y\) is called almost soft \(\beta\)-open if \(f((K, A)) \subset \sint(scl(f((K, A))))\) for all soft \(\beta\)-open set \((K, A)\) in \(X\).

**Theorem 4.5.** If \(f : X \to Y\) is almost soft \(\beta\)-open and weakly soft \(\beta\)-continuous functions, then \(f\) is a.s. \(\beta\)-continuous.

**Proof.**

Let \(x \in X\) and \((G, A)\) be a soft open set in \(Y\) containing \(f(x)\). Since \(f\) is weakly soft \(\beta\)-continuous, there exists \((K, A)\) such that \(f((K, A)) \subset scl((G, A))\). Since \(f\) is almost soft \(\beta\)-open, \(f((K, A)) \subset \sint(scl(f((K, A)))) \subset \sint(scl((G, A)))\). Hence \(f\) is a.s. \(\beta\)-continuous.

**Definition 4.6.** Let \(X\) be a soft space, then \(X\) is called...
1. **Almost soft regular** [5] if for any soft regular closed set \((F, A)\) of \(X\) and any point \(x \in X - (F, A)\) there exists disjoint soft open sets \((K, A)\) and \((G, A)\) such that \(x \in (K, A)\) and \((F, A) \sqsupseteq (G, A)\).

2. **Soft semi-regular** [3] is for any soft open set \((K, A)\) of \(X\) and each point \(x \in (K, A)\) there exists a soft regular open set \((G, A)\) of \(X\) such that \(x \in (G, A) \sqsubseteq (U, A)\).

**Theorem 4.7.** Let \((X, \tau, A)\) be a soft topological space. Then the following are equivalent:

1. \(X\) is almost soft regular
2. For each soft regular closed set \((G, A)\) and \(\forall x \in X - (G, A)\), there exists disjoint soft regular open sets \((F, A), (H, A)\) such that \(x \in (F, A)\) and \((G, A) \sqsubseteq (H, A)\).
3. For each soft regular open set \((H, A)\) of \(X\) and \(x \in (H, A)\), there exists a soft regular open set \((F, A)\) such that \(x \in (F, A) \sqsupseteq \text{scl}((F, A)) \sqsubseteq (H, A)\).

**Theorem 4.8.** If \(f : X \rightarrow Y\) is a weaklysoft\(\beta\)-continuous functions and \(Y\) is almost soft regular. Then \(f\) is a.s.\(\beta\).c.

**Proof.**
Let \(x \in X\) and \((G, A)\) be a softopen set of \(Y\) containing \(f(x)\). By the almost soft regularity of \(Y\), there exists a soft regular open set \((F, A)\) of \(Y\) such that \(f(x) \in (F, A) \sqsupseteq \text{scl}((F, A)) \sqsubseteq \text{sint}((F, A))\) by Theorem 4.7. Since \(f((U, A)) \sqsubseteq \text{scl}((F, A)) \sqsubseteq \text{sint}((F, A))\) we have \(f\) is a.s.\(\beta\).c.

**Theorem 4.9.** If \(f : X \rightarrow Y\) is an a.s.\(\beta\).c. function and \(Y\) is soft semi-regular. Then \(f\) is soft \(\beta\)-continuous.

**Proof.**
Let \(x \in X\) and let \((G, A)\) be a soft open set of \(Y\) containing \(f(x)\). By the soft semi-regularity of \(Y\), there exists a soft regular open set \((F, A)\) of \(Y\) such that \(f(x) \in (F, A) \sqsubseteq (G, A)\). Since \(f\) is a.s.\(\beta\).c., there exists \((K, A) \in S\beta\text{OS}(X, x)\) such that \(f((K, A)) \sqsubseteq \text{sint}((F, A)) = (F, A) \sqsubseteq (G, A)\) and hence \(f\) is soft \(\beta\)-continuous.

**Corollary 4.10.** If \(Y\) is a soft regular space, the following are equivalent for a function \(f : X \rightarrow Y:\)

1. \(f\) is soft \(\beta\)-continuous;
2. \(f\) is a.s.\(\beta\).c.;
3. \(f\) is weakly soft \(\beta\)-continuous.

**Proof.**
This proof form (Theorem 4.8 and Theorem 4.9).

**Definition 4.11.** The soft \(\beta\)-frontier of a soft subset \((F, A)\) of \(X\), denoted by \(s\beta\text{Fr}((F, A))\), is defined by \(s\beta\text{Fr}((F, A)) = s\beta\text{cl}((F, A)) \cap s\beta\text{cl}(X - (F, A)) = s\beta\text{cl}((F, A)) - s\beta\text{int}((F, A))\).

**Theorem 4.12.** The soft set of all points \(x\) of \(X\) at which a soft function \(f : X \rightarrow Y\) is not a.s.\(\beta\).c. is identical with the union of the soft \(\beta\)-frontiers of the inverse image of soft regular open set containing \(f(x)\).

**Proof.**
Let \(x\) be a point of \(X\) at which \(f\) is not a.s.\(\beta\).c. Then, there exists a soft regular open set \((G, A)\) of \(Y\) containing \(f(x)\) such that \((U, A) \cap (X - f^{-1}((G, A))) \neq \phi\) for all \((K, A) \in S\beta\text{OS}(X, x)\). Therefore, we have...
\( x \in s_{bc}l(X - f^{-1}((G, A))) = X - s_{bc}nt(f^{-1}((G, A))) \). Thus, we obtain
\( x \in s_{bc}Fr(f^{-1}((G, A))) \). Conversely, suppose that \( f \) is a.s.\( \beta \).c. at \( x \in X \) and let \((G, A)\) be a soft
regular open set containing \( f(x) \). Then there exists \((K, A) \in S_{bc}OS(X, x)\) such that \((K, A) \subseteq f^{-1}((G, A))\);
hence \( x \in s_{bc}nt(f^{-1}((G, A))) \). Therefore, it follows from theorem that \( x \in X - s_{bc}Fr(f^{-1}((G, A))) \). This completes the proof.

**Definition 4.13.** A function \( f : X \to Y \) is said to be

1. Complementary almost soft \( \beta \)-continuous if for each soft regular open set \((G, A)\) of \(Y\), \( f^{-1}(sFr((G, A)))\) is soft \( \beta \)-closed in \(X\), where \(sFr((G, A))\) denoted the soft frontier of \((G, A)\);

2. Weakly soft \( \alpha \) -continuous if for each point \( x \in X \) and each soft open set \((G, A)\) of \(Y\) containing \( f(x) \), there exists \((U, A) \in S_{ac}OS(X, x)\) such that \( f((U, A)) \subseteq scl((G, A)) \).

**Theorem 4.14.** If \( f : X \to Y \) is weakly soft \( \alpha \) -continuous and complementary almost soft \( \beta \) -continuous, then \( f \) is a.s.\( \beta \).c.

**Proof.**

Let \( x \in X \) and let \((G, A)\) be a soft regular open set of \(Y\) containing \( f(x) \). Then \( f(x) \in Y - sFr((G, A)) \) and hence \( x \in X - f^{-1}(sFr((G, A))) \). Since \( f \) is weakly soft \( \alpha \) -continuous, there exists
\((F, A) \in S_{ac}OS(X, x)\) such that \( f((F, A)) \subseteq scl((G, A)) \). Put \((U, A) = (F, A) \cap (X - f^{-1}(sFr((G, A)))) \). Then \((U, A) \in S_{bc}OS(X, x)\) by Corollary 2.5, and
\( f((U, A)) \subseteq f((F, A)) \cap (Y - sFr((G, A))) \subseteq scl((G, A)) \cap (Y - sFr((G, A))) = (G, A) \). This shows that \( f \) is a.s.\( \beta \).c.

**Theorem 4.15.** If \( f : X \to Y \) is a.s.\( \beta \).c., \( g : X \to Y \) is weakly soft \( \alpha \) -continuous and \( Y \) is soft \( T_2 \) -space, then the soft set \( \{ x \in X : f(x) = g(x) \} \) is soft \( \beta \)-closed in \(X\).

**Proof.**

Let \((K, A) = \{ x \in X : f(x) = g(x) \} \) and \( x \in X - (K, A) \). Then \( f(x) \neq g(x) \). Since \( Y \) is soft \( T_2 \) -space, there exists soft open sets \((G, A)\) and \((H, A)\) of \( Y \) such that \( f(x) \in (G, A) \), \( g(x) \in (H, A) \) and
\((G, A) \cap (H, A) = \emptyset \); hence \( \text{sint}(scl((G, A))) \cap scl((H, A)) = \emptyset \). Since \( f \) is a.s.\( \beta \).c., there exists
\((F, A) \in S_{bc}OS(X, x)\) such that \( f((F, A)) \subseteq \text{sint}(scl((G, A))) \). Since \( g \) is weakly soft \( \alpha \) -continuous, there exists a soft \( \alpha \)-open set \((L, A)\) of \( X \) containing \( x \) such that \( g((L, A)) \subseteq scl((H, A)) \). Now, put \((U, A) = (F, A) \cap (L, A) \), then \((U, A) \in S_{bc}OS(X, x)\) and
\( f((U, A)) \subseteq f((F, A)) \cap g((L, A)) \subseteq \text{sint}(scl((G, A))) \cap scl((H, A)) = \emptyset \).

Therefore, we obtain \((U, A) \cap (K, A) = \emptyset \) and hence \( x \in X - s_{bc}cl((K, A)) \). This shows that \((K, A)\) is \( \beta \)-closed in \(X\).

**Theorem 4.16.** If \( f_1 : X_1 \to Y \) is weakly soft \( \beta \) -continuous \( f_2 : X_2 \to Y \) is a.s.\( \beta \).c. and \( Y \) is soft \( T_2 \) -space, then the set \( \{ (x_1, x_2) \in X_1 \times X_2 : f_1(x_1) = f_2(x_2) \} \) is soft \( \beta \)-closed in \(X_1 \times X_2\).

**Proof.**

Let \((K, A) = \{ (x_1, x_2) \in X_1 \times X_2 : f_1(x_1) = f_2(x_2) \} \) and \( (x_1, x_2) \in X_1 \times X_2 - (K, A) \). Then \( f_1(x_1) \neq f_2(x_2) \) and there exists soft open set \((G_1, A)\) and \((G_2, A)\) of \( Y \) such that \( f_1(x_1) \in (G_1, A) \), \( f_2(x_2) \in (G_2, A) \) and
\((G_1, A) \cap (G_2, A) = \emptyset \); hence \( scl((G_1, A)) \cap sint(scl((G_2, A))) = \emptyset \). Since \( f_1 \) (resp. \( f_2 \)) is weakly soft \( \beta \) -continuous (resp. a.s.\( \beta \).c.), there exists \((U_1, A) \in S_{bc}OS(X_1, x_1) \) such that
\( f_1((U_1, A)) \subseteq scl((G_1, A)) \) (resp. \( f_2((U_2, A)) \in S_{bc}OS(X_2, x_2) \) such that...
\( f_2((U_2, A)) \bar{\subset} \sin(scl((G_2, A))) \).

Therefore, we obtain \((x_1, x_2) \in (U_1, A) \times (U_2, A) \bar{\subset} X_1 \times X_2 - (K, A) \) and 
\((U_1, A) \times (U_2, A) \in S_{\beta}OS(X_1 \times X_2) \). This shows that \((K, A)\) is soft \(\beta\)-closed in \(X_1 \times X_2\).

**Definition 4.17.** A soft topological space \((X, \tau, A)\) is said to be soft \(\beta\)-connected if \(X\) can not expressed as a union of two non-empty and disjoint soft \(\beta\)-open sets of \((X, \tau, A)\).

**Definition 4.18.** A soft space \(X\) is said to be soft \(\beta-T_2\) if for any distinct point \(x, y\) of \(X\), there exists disjoint soft \(\beta\)-open sets \((U, A), (G, A)\) of \(X\) such that \(x \in (U, A)\) and \(y \in (G, A)\).

**Theorem 4.19.** If for each pair of distinct points \(x_1, x_2\) in a soft space \(X\), there exists a function \(f\) of \(X\) into a soft \(T_2\)-space \(Y\) such that
1. \(f(x_1) \neq f(x_2)\);
2. \(f\) is weakly soft \(\beta\)-continuous at \(x_i\);
3. \(a.s.\beta.c.\) at \(x_2\). Then \(X\) is soft \(\beta-T_2\).

**Proof.**

Since \(X\) is soft \(T_2\)-space, there exists soft open sets \((G_1, A)\) and \((G_2, A)\) of \(X\) such that 
\(f_1(x_1) \in (G_1, A), f_2(x_2) \in (G_2, A)\) and 
\((G_1, A) \bar{\cap} (G_2, A) = \phi\); hence 
\(scl(((G_1, A)) \bar{\subset} \sin(scl(((G_2, A))) = \phi\). Since \(f\) is weakly soft \(\beta\)-continuous at \(x_1\), there exists 
\((U_1, A) \in S_{\beta}OS(X, x_1)\) such that 
\(f(((U_1, A)) \bar{\subset} scl(((G_1, A)))\). Since \(f\) is \(a.s.\beta.c.\) at \(x_2\), there exists 
\((U_2, A) \in S_{\beta}OS(X, x_2)\) such that 
\(f(((U_2, A)) \bar{\subset} scl(((G_2, A)))\). Therefore, we obtain 
\((U_1, A) \bar{\cap} (U_2, A) = \phi\). This shows that \(X\) is soft \(\beta-T_2\).

**Definition 4.20.** A function \(f : X \rightarrow Y\) has a soft \(\beta\)-closed graph if for each \((x, y) \in X \times Y - (F, A)(f)\), there exists 
\((U, A) \in S_{\beta}OS(X, x)\) and a soft open set \((G, A)\) of \(Y\) containing \(y\) such that 
\([(U, A) \times cl(((G, A)))] \bar{\cap} (F, A)(f) = \phi\).

**Lemma 4.21.** A function \(f : X \rightarrow Y\) has a soft \(\beta\)-closed graph if and only if \(\forall (x, y) \in X \times Y\) such that 
\(y \neq f(x)\), there exists a soft \(\beta\)-open set \((K, A)\) and any soft open set \((G, A)\) containing \(x\) and \(y\), respectively, such that 
\(f((K, A)) \bar{\cap} scl((G, A)) = \phi\).

**Theorem 4.22.** If \(f : X \rightarrow Y\) is an \(a.s.\beta.c.\) function and \(Y\) is soft \(T_2\)-space, then \(f\) has a soft \(\beta\)-closed graph.

**Proof.**

Let \((x, y) \in X \times Y\) such that \(y \neq f(x)\). Then there exists soft open sets \((G, A)\) and \((H, A)\) such that 
\(f(x) \in (G, A), y \in (H, A)\) and 
\((G, A) \bar{\cap} (H, A) = \phi\); hence 
\((G, A) \bar{\subset} scl((H, A)) = \phi\). Then 
\(f(x) \in Y - scl((H, A))\) and 
\(Y - scl((H, A))\) is soft regular open in \(Y\). There exists 
\((U, A) \in S_{\beta}OS(X, x)\) such that 
\(f(((U, A)) \bar{\subset} Y - scl((H, A))\) and hence 
\(f(((U, A)) \bar{\subset} cl(((H, A)) = \phi\). Therefore; by Lemma 4.21 \(f\) has a soft \(\beta\)-closed graph.

**Corollary 4.23.** If \(f : X \rightarrow Y\) is a soft \(\beta\)-continuous function and \(Y\) is soft \(T_2\)-space, then \(f\) has a soft \(\beta\)-closed graph.

**References**


[23] T. Noiri and V. Papa, "Properties of \( \beta \)-continuous functions (submitted)."


